University of the Witwatersrand

Advanced Techniques in Physics : 2002

Examination : June 2002

Instructions

1. There are six questions in total, grouped into two sections. The two sections must be answered in separate books.
2. Section 1 comprises of questions 1, 2 and 3.
3. Section 2 comprises of questions 4 and 5. Section two also contains a take - home question. This will be question 6 which does not appear in this paper and will not be answered in the answer book. You will receive question 6 on Wednesday 8:30 27^{th} June. There will be 24 hours to answer question 6.
4. Answer all questions, 1 to 5. Questions can be attempted in any order. Start each new question on a new page.

Time:	Questions 1 to 5	$2\frac{1}{2}$ hours	(exam conditions)
			Total Marks (1-5) = 130
	Question 6	One day	(take home conditions)
			Total Marks (6) = 70

Questions 1-3 not included.

4. Consider the ordinary differential equation

$$\frac{d^2y}{dx^2} + k^2(x)y = S(x)$$

where initial values are specified, and k^2 may have either sign.

(a) Discuss the condition (ill- or otherwise) of an attempted numerical integration of this O.D.E. by, for example, a Runge-Kutta method.

(6)

(b) Relate the above discussion to the condition of numerical generation of special functions by recurrence relations. As a specific example consider recurrence relation for the cylindrical Bessel functions

$$C_{n-1}(x) + C_{n+1}(x) = \frac{2n}{x} C_n(x)$$

C_0 and C_1 are replaced by J_0, J_1 or Y_0, Y_1 for the regular or irregular solution respectively. Demonstrate (using the recurrence relation) why forward recurrence is stable for generation of the Y_n but not for generation of the J_n unless $n < x$.

(6)

(c) Describe an alternative method for generation of the J_n when $n > x$.

(3)

5. In discussing computer representation of numbers and arithmetic we can write :-

true	stored
a	$x = a(1+\delta_1)$
b	$y = b(1+\delta_2)$
$a \circ b$	$x \hat{\circ} y = x \circ y(1+\delta_3)$

The symbol $\circ$ represents any primary operation.

a) Find the relative error in addition to first order and show that if a and b differ in sign with $a + b \approx 0$ then there is a disasterous loss of accuracy.

(8)

b) Show in addition that error propogation in summing truncated series is minimised by adding terms in ascending order of magnitude.

(7)

6. To be a take home component on Friday 21^{st} June.

(70)

Total **Question 1-5 : [130 marks]**