University of the Witwatersrand
Advanced Techniques in Physics : 2003
Examination : June 2003

Instructions

1. There are six questions in total, grouped into two sections. The two sections must be answered in separate books.
2. Section 1 comprises of questions 1, 2 and 3.
3. Section 2 comprises of questions 4 and 5. Section two also contains a take - home question. This will be question 6 which does not appear in this paper and will not be answered in the answer book. You will receive question 6 on Friday 8:30 20^{th} June. There will be 24 hours to answer question 6.
4. Answer all questions, 1 to 5. Questions can be attempted in any order. Start each new question on a new page.

Time:	Questions 1 to 5	$2\frac{1}{2}$ hours	(exam conditions)
			Total Marks (1-5) = 130
	Question 6	One day	(take home conditions)
			Total Marks (6) = 70

1. Section 1 Question 1

   a) Section 1 Question 1a (10)

   b) Section 1 Question 1b (10)

   c) Section 1 Question 1c (10)

Total for Question 1 [30]

2. Section 1 Question 2 Total for Question 2 [30]

3. Section 1 Question 3 Total for Question 3 [40]

4. With respect to Gaussian Quadratures,

   a) Derive the result

$$\int_a^b f(x)\,dx = \sum_{k=0}^{m} a_k \overline{f}(x_k)$$

   where $\overline{f}(x)$ is a polynomial interpolating the integrand $f(x)$. (10)

   b) Discuss the order of Gaussian quadratures w.r.t. the Newton-Cotes formulae. (5)

   c) Discuss the relationship between higher order and higher accuracy. How may these considerations be accomodated in the scheme of Gaussian quadratures ? (5)

Total for Question 4 [20]

5. Finite difference representations discretising P.D.E. operators are the starting point for numerical methods of solution.

   (a) Write down the finite difference formula for the Laplacian for a scalar function of two variables. Use the Taylor expansion to demonstrate explicitly the largest error term and its order. (5)

   (b) If ϕ_1 and ϕ_2 are approximate solutions corresponding to mesh sizes h and h/j respectively, then show that $(j^2\phi_2 - \phi_1)/(j^2 - 1)$ is an improved solution. (5)

Total for Question 5 [10]

6. To be a take home component on Friday 20^{th} June.

Total for Question 6 [70]

Total **Question 1-5 : [130 marks]**