University of the Witwatersrand
Advanced Techniques in Physics : 2003
Examination : June 2003

Instructions
This is the take - home question. It is to be handed out on Friday 8:30 20th June.
There will be 24 hours to answer question 6.
Time for Question 6 : One day (take home conditions) Total Marks (Q6) = 70

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6. This question requires some programming.
Please remember to hand in your diskette with your script.

The first few years of LEP running were devoted to the study of Z production and decay with the energy of the accelerator being varied in small energy steps around the resonance (Z invariant mass).

One of the important results has been the determination of a the number of lepton families to a certain confidence limit. Figure **??** illustrates this point. One can see that only the theory for three families of particles correctly describes the data.

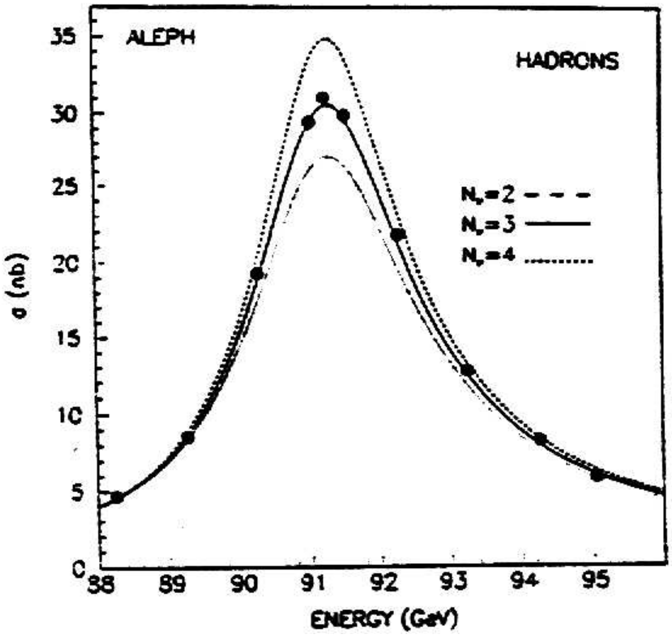

Figure 1: ALEPH results for hadronic decays of Z.

In this project, you will have to verify the conclusion that only three families of particles describe the data, and in addition, determine the confidence limits of this statement. You will also have to consider the assumptions underlying this conclusion, so that you can comment on how reliable it can be expected to be.

The absolute cross-section (probability) for the processes

$$e^+e^- \longrightarrow f\bar{f}$$

is recorded near the resonance energy m_Z for Z production where the dominant term is in fact Z exchange.

In the three family model, f is either a lepton: e, μ or τ, a neutrino: ν_e, ν_μ or ν_τ, or one of the five quark flavours: u, d, s, c or b. (Only fermions with masses $m_f < m_Z/2$ contribute directly to the Z decay cross-section.)

The theoretical calculation of the cross-section at the lowest order is done using the first Born approximation. At this level the resonance cross-section is a Breit-Wigner Lorentzian. Without the radiative corrections, however, the predicted line shape is grossly inadequate. The radiative corrections modify the predictions for the partial widths, and introduce an energy dependence in the resonance width. The detailed shape of the resonance is therefore sensitive to many aspects of the Standard Model. This includes constraining the top mass, QCD checks, measurements of α (the fine structure constant), setting lower limits on the Higgs mass and so on. Out of this wealth of information available in this data, for our purposes we are only interested in measuring, with confidence limits, the number of particle families.

Considering the above, the new form for the resonance cross-section becomes :-

$$\sigma_f(s) = \frac{12\pi s}{m_Z^2}\frac{\Gamma_e\Gamma_f}{|s - m_Z^2 + is\Gamma_Z/m_Z|^2}$$

As usual, Γ_Z is the total width, which is a direct sum of the partial widths for each possible exit channel. Γ_e accounts for the formation of the Z in the entrance channel of an $e^+ - e^-$ collider. Γ_f represents the partial width for a particular fermion f in the exit channel. This is selected by choosing the appropriate subset of events in the data set.

A significant correction (up to 30%) at the resonance energy, is left out of this expression. The electron-positron pair in the entrance channel bremsstrahl photons which lower the centre of mass energy available for Z production. This causes a high energy tail as well as a lowering of the peak in the resonance cross-section. Consider the raw data of figure **??** and convince yourself that you see this effect. The Breit-Wigner Lorentzian line shape given above would therefore not be adequate, and would require modification in order to correctly model the observed line shape. These modifications have been computed, and are available in the literature.

From the LEP results it can be well established that there are no charged leptons from possible additional families with masses $m_f < m_Z/2$, since these would have been seen.

There could conceivably be many families (quarks and charged leptons). However, due to the condition $m_f > m_Z/2$, only the neutrinos of these other families are usually expected to contribute to the total Z width, Γ_Z. The reasoning, in the case of a possible fourth particle family, is that the lepton and quarks for this family may be massive enough not to appear in the calculation of the Z line shape. However, the neutrino of this possible fourth family is unlikely to be massive enough to be excluded (neutrinos are at least rather light).

$$\Gamma_Z = \Gamma_h + 3\Gamma_l + N_v\Gamma_v$$

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Γ_h is the total hadronic width, Γ_l is the width due to each of the charged lepton channels and Γ_ν is the width for each neutrino channel. If there are more than three families, then $N_\nu > 3$

Electroweak predictions for the relative rates of the channels are as follows:

each neutrino channel	6.7%
each charged lepton channel	3.35%
total hadronic channels	70%

From this it can be seen that each additional neutrino channel will increase Γ_Z by 6.7%. The Z decay events can be observed and sorted according to the exit channel represented by $f\bar{f}$ separately, except for the neutrino exit channel. Hadronic data will of course have the better statistics. Hadronic data (one selects a subset of the observed events where the primary particles in the exit channel are hadrons) is therefore used to provided the most stringent evaluation of the number of fermion families. Hadronic data for Z production is reproduced in the table below. It has however been doctored. The effect of entrance channel bremsstrahlung (discussed above) has been unfolded from the raw data according to a model for it, so that the simple form of $\sigma_f(s)$ presented above (which is not corrected for entrance channel bremsstrahlung) can nevertheless still be used. This has been done by the examiner to simplify the question. In practise, one prefers not to doctor data, and rather use a more complex theory in the description of the Z line shape.

Energy ($\sqrt{s}$) (GeV)	Cross-section ($\sigma_h(s)$) (nb)	absolute error (nb)
88.18	6.10	0.10
89.18	11.41	0.20
90.18	24.79	0.44
90.9	40.08	0.47
91.18	41.75	0.75
91.5	38.29	0.68
92.18	25.37	0.45
93.18	11.95	0.21
94.18	6.41	0.11
95.18	3.70	0.07

Table 1: Table of ALEPH results for hadronic decays of Z - (unfolded). Based on : J. Steinberger, Phys. Rep. 203 (1991) 345-381.

a) The fermion cross-section $\sigma_f(s)$ above can be specialised to the hadronic cross-section, $\sigma_h(s)$ and $\Gamma_f = \Gamma_h$. On resonance, we will then find

$$\sigma_h(s = m_Z^2) = \frac{12\pi\Gamma_e\Gamma_h}{m_Z^2\Gamma_Z^2}$$

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This can be re-inserted into the energy dependent hadronic expression as

$$\sigma_h(s) = \frac{\sigma_h(s = m_Z^2) s \Gamma_Z^2}{|s - m_Z^2 + is\Gamma_Z/m_Z|^2}$$

The hadronic data can now be fitted with only m_Z, $\sigma_h(s = m_Z^2)$ and Γ_Z as free parameters. Perform this fit using the assumption of three particle families and extract values and errors for these parameters. You should use the Levenberg-Marquardt method *mrqmin* in Numerical Recipes. You should be aware of the discussion and examples provided in the text and in the electronic distribution. You should also convert the expression to real arithmetic only.
(Hint: You would be wise to include a constant background under the curve, to model the non-resonant portion of the cross-section, as well as any other systematic effects.) (35)

b) This first fit of the previous question represents the case of only three families, and it therefore determines Γ_Z for $N_v = 3$. Now perform additional fits where you imagine first two particle families and then four particle families. You do this by fixing the values of $\sigma_h(s = m_Z^2)$ and Γ_Z, consistent with the number of families under consideration (use the formulae given above). For example, when considering the case of $N_v = 4$, then increase Γ_Z by the weak interaction theory prediction of 6.7% for an additional neutrino exit channel. Use this new value of Γ_Z to predict $\sigma_h(s = m_Z^2)$. You will then be doing a fit with only one free parameter for the lineshape, and also a free parameter for the background. Present your results in a table, with columns for : the number of particle families being considered, the three line shape parameters, the number of degrees of freedom in each fit, the Chi-squared value for the fit and the reduced Chi-squared value. (10)

c) Produce output similar to that shown in figure **??**, except that your plot has the electron bremsstrahlung corrections unfolded from both the data and theory. (10)

d) Perform an F-test of significance on your result. Let us call the distribution of the data around the three particle family *distribution 1.* Then the distribution of the data around the other assumption (two or four particle families) is *distribution 2.* The null hypothesis maintains that the variance of *distribution 1* is the same as the variance of *distribution 2* (We hope in fact the variance of *distribution 2* is much bigger than that of *distribution 1*). The F-statistic is formed by the ratio of the reduced Chi-squareds for the two distributions, the one favoured by the null hypothesis in the denominator.

$$F_{exp}(\nu_1, \nu_2) = \frac{\chi^2_{\nu_2}}{\chi^2_{\nu_1}}$$

Clearly, the F-statistic will be large if we are to reject the null-hypothesis. As discussed in the lecture notes, the F-statistic is well defined and belongs to a distribution which can be calculated. The cumulative F-distribution formed by integrating the F-distribution up to a given point F_{CL} is $Q(F_{CL}; \nu_1, \nu_2)$. The CL point expressed as a percentage is the Confidence Level. If the null hypothesis were correct, then to a given CL, F_{exp} could lie anywhere between 0 and F_{CL}, with the difference from the required value of $F_{exp} = 1$ being justified by normal statistical variation in the data sets. However, if $F_{exp} > F_{CL}$, then this would mean the null hypothesis is rejected at this CL. Find the appropriate chapter of Numerical Recipes that generates the cumulate F-distribution. Be advised to think through the discussion carefully to note that the Numerical Recipes routine in fact generates the compliment of $Q(F_{CL}; \nu_1, \nu_2)$. Using your your tabulated results of question 6 c), Find the maximum CL for choosing three particle families above two and four particle families, based on the LEP data. (10)

e) Discuss the assumptions inherent in determining parameters, errors and confidence limits from experimental data in this way. This data represents 175000 Z decay events. Half the events are shared equally between the three points near the maximum, and half are shared equally among the remaining events. Assume the luminosity events, for absolute calibration, have similar statistics to the hadron events. Determine the number of events when the confidence limit was / will be 99%. (5)

(The diskette to be handed in must contain : A well documented tested source code file and a graphic file for display of the relevant plots.)

Total for Question 6 [70]