University of the Witwatersrand
Physics IIE (Engineering) : PHYS284 - 2002
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## Class Test 1 — 60 min.

## Special Relativity & Intro to Quantum Mechanics

1. (a) Given that the Lorentz Transformation is :

$$\begin{aligned} x' &= \frac{(x - vt)}{\sqrt{1 - v^2/c^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{(t - \frac{vx}{c^2})}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

derive the formula for the relativistic time dilation

$$t = t_0/\sqrt{1 - v^2/c^2}.$$

(6)

(b) A spacecraft is moving relative to the earth. An observer on earth finds that, according to her clock, 3601s elapse between 1pm and 2pm on the spacecraft's clock. What is the spacecraft's speed relative to the earth ? (6)

(c) Show that the relativistic velocity transformation is

$$v'_x = \frac{v_x - v}{1 - \frac{v_x v}{c^2}}$$

(5)

(d) Two spaceships, A and B, are approaching each other from opposite directions. An observer on earth measures their velocities to be 0.750c and -0.850c respectively. What is their actual relative velocity, as would be measured by an observer in either ship. (5)

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2. Consider the "particle in a box" problem from a quantum mechanical perspective.

(a) Sketch the appropriate potential with labels. (5)

(b) Write down the wave function which is the general solution of Schrödingers equation

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi$$

for the box interior. (2)

(c) Mention the boundary conditions (3)

(d) Apply these boundary conditions to deduce the quantisation condition for the energy of the particle in the box.

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

(6)

(e) Show that that this last statement is consistent with Heizenberg's Uncertainty Principle. (5)

(f) Can the particle in the box ever have no kinetic energy ? Explain why. (2)

(g) The final form of the wave-function may now be written

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{\sqrt{2mE_n}}{\hbar} x$$

Explain mathematically the procedure used to determine the value of the normalisation constant. (3)

(h) Indicate the integral that would need to be evaluated to determine the average position $\langle x \rangle$ of the particle in the box.

(2)

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Total Marks [50]