University of the Witwatersrand
Physics IIE (Engineering) : PHYS284 - 2003
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Class Test 1 — 60 min.

Special Relativity & Intro to Quantum Mechanics

1. Two electrons in a collider experiment are approaching each other from opposite directions. An observer in the lab measures their velocities to be 0.750c and -0.850c respectively. What is the velocity of the one electron as seen from the other ? (8)

2. (a) Suppose that a particle of mass m initially at rest has a force acting on it while it moves a distance s and derive the expression

$$E_k = \int_0^v v \, d(\gamma m_0 v)$$

for the work done on the particle to bring it to a final velocity v. (5)

   (b) Use integration by parts ($\int x\,dy = xy - \int y\,dx$) to derive the relativistic expression for the kinetic energy

$$E_k = mc^2 - m_0 c^2.$$

Hint : $\int \frac{x\,dx}{\sqrt{1-x^2}} = -(1-x^2)^{1/2}$ (5)

   (c) Name the two terms on the right hand side and discuss their significance. (4)

   (d) Evaluate the second term for an electron of mass 9.11×10^{-31} kg in units of electron volts. (5)

   (e) Suppose an electron were accelerated by a potential difference of 1.0 kV. Evaluate the Lorentz factor, γ, for the electron working from the expression for the kinetic energy of the electron derived above. Comment on whether such an electron would be regarded as relativistic or not. (5)

   (f) Show that the normal expression for the kinetic energy is recovered for low velocities. Hint : Binomial Expansion : $(1+x)^n \approx 1 + nx$ (8)

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3. Consider the "particle in a box" problem from a quantum mechanical perspective. The un-normalised wave function for a particle in a box is

$$\psi_n(x) = A \sin \frac{\sqrt{2mE_n}}{\hbar} x = A \sin \frac{n\pi x}{L} \qquad \text{n=1,2,3...}$$

   (a) The normalisation condition is chosen so that $|\psi(x)|^2$ represents a probability density. Perform this normalisation and show that the normalised wave function is

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{\sqrt{2mE_n}}{\hbar} x = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \qquad \text{n=1,2,3...}$$

Hint : $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ (6)

(b) Show, with a clearly argued derivation, that quantisation of the energy of the particle in a box arises naturally by application of the boundary conditions leading to the energy spectrum

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \text{n=1,2,3. . . .}$$

(6)

(c) Can the particle in the box ever have no kinetic energy ? Explain why. (2)

(d) Sketch the wave functions and the corresponding probability densities for the first three energy levels for the particle in the box. Label the diagram clearly. (6)

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Total Marks [60]