



Class Test 1

60 min. and 60 marks

Special Relativity & Intro to Quantum Mechanics

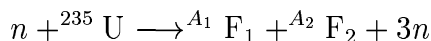
1. A muon is created in the upper atmosphere at a typical altitude of 6 km, due to cosmic ray interactions with the nuclei of atoms in the atmosphere. The velocity of the muon is about $v = 0.995c$. The lifetime of the muon is $2.2 \mu\text{s}$. Show the following :

- (a) Non-relativistic analysis leads to the conclusion that very few muons would survive long enough to reach the surface of the earth (sea level). (4)
- (b) In actual fact, experiment shows most of the muons reach the surface of the earth. Explain how this occurs considering the viewpoint of an observer on earth. (5)
- (c) Show that a hypothetical observer travelling with the muon would agree with the earth bound observer. (5)

(Hint : You will need to use time dilation in the one case, and length contraction in the other, but you must justify your choice, as well as perform the calculation.)

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2. (a) Sketch the relativistic and the classical expressions for kinetic energy on the same graph and label the graph as completely as you can. (5)
- (b) Show that the classical expression is recovered from the relativistic one at low velocities. (5)
- (c) In an atomic bomb, a uranium nucleus is induced to fission by neutron capture, leading to two fission fragments of roughly equal mass as well as a few neutrons.



- i. Which side of the equation would be lighter, considering this is an exothermic reaction. Explain your answer. (4)
- ii. The mass difference is about 21% of the mass of a nucleon. Calculate the energy release per fission and express the answer in MeV. (6)

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3. Use the Uncertainty Principle to show that if an electron were confined inside a nucleus of diameter $2 \times 10^{-15}\text{m}$, it would have to be moving relativistically, whereas a nucleon confined to move in the same nucleus would be moving non-relativistically

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4. A linear quantum dot can be modelled as a electron in a box. It can be formed by depositing contacts on a slab of GaAs and then applying a bias voltage in such a way as to create a linear confining potential.

(a) Suppose the observed ground-state electron energy is 0.03 eV. How big is the quantum dot ? (6)

(b) Find the expectation value $\langle x \rangle$ for the position of the electron. (6)

(Reminder : $P(x)dx = |\psi(x)|^2 dx$ and $\langle x \rangle = \int xP(x)dx$).

(c) Show that the ground state energy of a particle in a rectangular linear potential well with infinitely high sides is compatible with Heizenberg's Uncertainty Principal. (4)

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Note : Using $\hbar c = 197 \text{ MeV}\cdot\text{fm}$ with $1 \text{ fm} = 10^{-15}\text{m}$ can speed things up.

Total Marks [60]