University of the Witwatersrand

Physics IIE (Engineering) : PHYS284 - 2005
Course Co-ordinator : 1st Semester : Dr SH Connell

Class Test 1 **60 min. and 60 marks**

Special Relativity & Intro to Quantum Mechanics

1. (a) Starting from the expression for work done on a particle to accelerate it, show that

$$W = E_k = \int_0^v v \, d(\gamma m_0 v),$$

and hence derive the expression for kinetic energy

$$E_k = mc^2 - m_0c^2.$$

(7)

(b) Give the names of the two terms on the right of the last equation. (2)

(c) Show that this expression reduces to the normal classical expression $\frac{1}{2}m_0v^2$ at low speeds. (6)

(d) What are the implications for the law of conservation of energy ? (3)

(e) Given that the Lorentz Transformation is :

$$\begin{aligned} x' &= \frac{(x - vt)}{\sqrt{1 - v^2/c^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{(t - \frac{vx}{c^2})}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

derive the formula for the relativistic length contraction

$$L = L_0\sqrt{1 - v^2/c^2}.$$

(6)

(f) Derive an expression for the relativistic change in the density of the object. (6)

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2. Use the Uncertainty Principle to show that if an electron were confined inside a nucleus of diameter 2×10^{-15}m, it would have to be moving relativistically, whereas a nucleon confined to move in the same nucleus would be moving non-relativistically (10)

3. Consider the un-normalised wave function for a particle in a 1-D box.

$$\psi_n(x) = A \sin \frac{\sqrt{2mE_n}}{\hbar} x = A \sin \frac{n\pi x}{L}$$

   (a) Apply the normalisation requirement

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1$$

   to show that

$$A = \sqrt{\frac{2}{L}}.$$

   (10)

   (b) Find the probability that a particle trapped in a box L wide can be found between $0.45L$ and $0.55L$ for the ground state.
   (You may need : $2\sin^2 t = 1 - \cos 2t$.) (10)

[20]

Total Marks [60]