University of the Witwatersrand
Physics IIE (Engineering) : PHYS284 - 2006
Course Co-ordinator : 1st Semester : Dr SH Connell

Class Test 1 **60min. and 60 marks**

Special Relativity & Intro to Quantum Mechanics

1. Given that the Lorentz Transformation is :

$$\begin{aligned} x' &= \frac{(x - vt)}{\sqrt{1 - v^2/c^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{(t - \frac{vx}{c^2})}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

   (a) Show that the relativistic velocity transformation is

$$v'_x = \frac{v_x - v}{1 - \frac{v_x v}{c^2}}.$$

   (6)

   (b) Show that the speed of light is the same in all reference frames. (6)

   (c) Two spaceships, A and B, are approaching the international space station from opposite directions. The captain of the space station measures their velocities to be 0.750c and -0.850c respectively. What is their actual relative velocity, as would be measured by an observer in either ship. (6)

   (d) The Russian cosmonaut, Sergei Avdeyev, orbited Earth at 27000 km/h for a total of 748 days. Show that he aged less than his earthbound colleagues by about 1/50th of a second. (6)

   (e) Now suppose that Avdeyev travelled at 99.995 % of the speed of light to a star about 500 light years away and back. Show that he would have returned to an Earth 1000 years in the future but he would only have aged by ten years. (6)

   [30]

2. Consider the "particle in a box" problem from a quantum mechanical perspective.

   (a) Sketch the appropriate potential with labels. (5)

   (b) Write down the wave function which is the general solution of Schrödingers equation

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0$$

   for the box interior. (3)

   (c) Mention the boundary conditions (3)

(d) Apply these boundary conditions to deduce the quantisation condition for the energy of the particle in the box.

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

(6)

(e) Show that that this last statement is consistent with Heizenberg's Uncertainty Principle. (5)

(f) Can the particle in the box ever have no kinetic energy ? Explain why. (2)

(g) The wave-function may now be written

$$\psi_n(x) = A\sin\frac{\sqrt{2mE_n}}{\hbar}x = A\sin\frac{n\pi x}{L}$$

Determine the value of the normalisation constant. (6)

[30]

Total Marks [60]