University of the Witwatersrand
Physics IIE (Engineering) : PHYS2007 - 2007
Course Co-ordinator : 1st Semester : Dr SH Connell

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**Class Test 1** **60min. and 60 marks**

## Special Relativity

1. Starting from

$$W = E_k = \int_0^s F ds = \int_0^s \frac{d(mv)}{dt}\, ds$$

(a) Show that the relativistic expression for the kinetic energy is

$$E_k = mc^2 - m_0c^2.$$

(8)

(b) State the physical meaning of the two terms on the right hand side of the previous expression. (2)

(c) Show that the non-relativistic expression can be recovered at low velocities. (5)

(d) Sketch a graph of kinetic energy versus velocity relative to the speed of light for both the relativistic and non-relativistic expressions and label completely. (5)

[20]

2. A new synchrotron facility called the Large Hadron Collider (LHC) is being installed at the CERN experimental physics complex near Geneva. The ring has a circumference of 27km, and is due for commissioning starting in November this year. It will be able to accelerate protons to a kinetic energy of 7 TeV (1 Tera eV = $10^{12}$ eV).

(a) Show that the rest mass of the proton in units of electron volts is 511 keV.
(The mass of the proton is $1.673 \times 10^{-27}$ kg.) (3)

(b) Calculate the total energy of the accelerated proton and then show that the relativistic $\gamma$-factor is greater than $7 \times 10^3$.
(Hint: work from the relativistic energy expression.) (3)

(c) Show that the velocity of the proton is almost that of the speed of light, $v_p/c \approx (1-\delta)$ where $\delta = 9 \times 10^{-9}$.
(Hint: you may need the binomial approximation to first order.) (5)

(d) What will be the mass (in units of kg) of such a high energy proton as seen by a scientist in the lab. (2)

(e) Calculate the time taken for a round trip of the proton as seen by a scientist in the lab and for a observer in a frame fixed on the accelerated proton. (4)

(f) What is the ring circumference for an observer in a frame fixed on the accelerated proton. (2)

(g) In some of the experiments at the LHC, there will be two counter-rotating beams. The one beam is composed of 7 TeV protons circulating in one direction and the other beam is composed of 7 TeV anti-protons circulating in the opposite direction. The beams will be brought to specific collision points, where large detectors will record the results of the various interactions that will occur.

i. Calculate the total collision velocity from the perspective of Galilean Relativity (2)

ii. Calculate the total collision velocity from the perspective of Special Relativity. (If you can show $v/c \approx 1 - \frac{1}{2}\delta^2$ by using the binomial expansion carefully where necessry, then 3 marks extra). (4)

[25]

3. The Global Positioning System (GPS) consists of satellites with orbital speeds of about 3.9 km/s in a frame centred on the Earth. The orbital radius of the satellites is about 26,600 km.

(a) Do the satellite bound clocks tick faster or slower than the earth bound clocks, considering only the effects of Special Relativity ? (2)

(b) If 12 hours have passed on the satellite, what would the elapsed time have been on the earth, considering the effects of Special Relativity ? Express your answer as a time difference in microseconds.
(Use an approximate method if your calculator has insufficient accuracy.) (5)

(c) What position error would this represent, if the corrections were not made ? (4)

(d) What procedures are implemented to avoid this error ? (4)

Total for Question 1 [15]

Total Marks [60]