University of the Witwatersrand
Physics IIE (Engineering) : PHYS284 - 2002
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Quantum Mechanics of Atoms, Statistical Mechanics, Solid State Physics, Semiconductivity

Class Test 2 — 60 min.

1. (a) From the table of the normalised wave functions for the hydrogen atom, write down the wave function for the $2p$ electron with no z-component of orbital angular momentum. (2)

   (b) Taking note that the angular parts of the wavefunction are separately normalised, show that the expression for the electron's radial probability distribution is $P(r)dr = \frac{r^4}{8a_0^5}e^{-r/a_0}$.
   Note that the volume element in spherical co-ordinates is $dV = r^2 \sin\theta dr d\theta d\phi$ and that $\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$.
   (4)

   (c) Sketch this radial probability distribution.
   (4)

   (d) Show that the most probable value of r for this electron is $4a_0$.
   (5)

   [15]

2. The density of aluminium (group III of the Periodic Table) is 2.70 g/cm^3 and its atomic mass is 26.97 u.

   (a) Calculate the Fermi energy in aluminium. (6)

   (b) The measured Fermi energy is 11.8 eV. Cind the effectiveg mass of a conduction electron in aluminium. (6)

   (c) Explain the physical origins of effective mass. (6)

   (d) Show that electronic density of states

   $$n(\epsilon)d\epsilon = \frac{8\sqrt{2}\pi V m^{3/2}}{h^3}\frac{\sqrt{\epsilon}d\epsilon}{e^{(\epsilon-\epsilon_F)/kT}+1}$$

   may be expressed as

   $$n(\epsilon)d\epsilon = (3N/2)\epsilon_F^{-3/2}\sqrt{\epsilon}d\epsilon$$

   at 0 K. (6)

   (e) Now show that the total energy of the electron gass at 0 K is

   $$E_0 = \frac{3}{5}N\epsilon_F$$

by evaluating

$$E_0 = \int_0^{\epsilon_F} \epsilon n(\epsilon) d\epsilon.$$

and hence arrive at the result

$$\overline{\epsilon}_0 = \frac{3}{5} N \epsilon_F$$

(6)

(f) If electrons behaved classically, what would the temperature of the metal have to be for a simialr average energy to of the electrons to be realised. (6)

[24]

Total Marks [50]