University of the Witwatersrand
Physics IIE (Engineering) : PHYS284 - 2003
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Quantum Mechanics of Atoms, Statistical Mechanics, Solid State Physics, Semiconductivity

Class Test 2 — 60 min. (60 marks)

1. (a) Derive the expression for the hydrogenic radial probability density

$$P(r)dr = r^2|R(r)|^2 dr$$

using the normal expression for obtaining probabilities from wave functions. The hydrogenic wave function is

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$$

and the volume element in spherical polar co-ordinates is

$$dV = r^2 \sin\theta dr\, d\theta\, d\phi.$$

(5)

(b) Show that the most probable value for r for the 2p electronic state is $4a_0$ (work from $P(r)dr$) using the appropriate wavefunction in the attached table. (7)

(c) How much more likely is a 1s electron in a hydrogen atom to be at a distance a_0 from the nucleus than $a_0/2$. (8)

[20]

2. (a) The number of fermions in a fermi gas (for example, electrons in a metal) that have energies from ϵ to $\epsilon + d\epsilon$ is

$$\begin{aligned} n(\epsilon)d\epsilon &= g(\epsilon)f_{FD}(\epsilon)d\epsilon \\ &= \frac{8\sqrt{2}\pi L^3 m^{3/2}}{h^3}\frac{\sqrt{\epsilon}d\epsilon}{e^{(\epsilon-\epsilon_F)/kT}+1} \end{aligned}$$

Consider N electrons in the low temperature limit where all the low-lying states are occupied. Show that the energy of the last filled state, known as the fermi energy, is given by

$$\epsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}.$$

(8)

(b) The density of metallic zinc is 7.13 g/cm^3 and the atomic mass of the zinc atom is 65.4 u. The Fermi energy in zinc metal is 11.0 eV. Work out the effective mass of a delocalised electron in Zinc metal. Express your answer in terms of the free electron mass. Why is there a difference ? (8)

[16]

3. (a) The velocity of an electron in a band is the group velocity of the wave packet $v_g = \frac{d\omega}{dk}$. The mass of the electron in the band is defined by Newton's 2nd Law of motion :- $F = m^* \frac{dv}{dt} = \frac{dp}{dt} = \frac{d(\hbar k)}{dt}$. Show then that the effective mass of the electron in a semiconductor is

$$m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}.$$

(8)

(b) In an intrinsic semiconductor, the number of electrons in the conduction band and holes in the valence band is given by the following expressions, respectively

$$n_e = 2V\left(\frac{m^*kT}{2\pi\hbar^2}\right)^{3/2} e^{(\epsilon_F - \epsilon_g)/kT} \qquad \text{and} \qquad n_h = 2V\left(\frac{m_h^*kT}{2\pi\hbar^2}\right)^{3/2} e^{-\epsilon_F/kT}.$$

Show that the Fermi level will be approximately in the middle of the band gap. (8)

(c) Using data for Gallium Arsenide,

$$\begin{aligned} m_e^* &= 0.072\ m_e \\ m_h^* &= 0.4\ m_e \\ \epsilon_g &= 1.0\ \text{eV}, \end{aligned}$$

verify that at room temperature, the Fermi-level is in the middle of the gap to within 7%. (8)

[24]

Total Marks [60]

n	l	m_l	$\Phi(\phi)$	$\Theta(\theta)$	$R(r)$	$\psi(r, \theta, \phi)$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{\pi}\, a_0^{3/2}} e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$*	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}\, a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos\theta$	$\frac{1}{2\sqrt{6}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \cos\theta$
2	1	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin\theta$	$\frac{1}{2\sqrt{6}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \sin\theta\, e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3}\, a_0^{3/2}} \left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right) e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi}\, a_0^{3/2}} \left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right) e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos\theta$	$\frac{4}{81\sqrt{6}\, a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi}\, a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \cos\theta$
3	1	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin\theta$	$\frac{4}{81\sqrt{6}\, a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \sin\theta\, e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4} (3\cos^2\theta - 1)$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} (3\cos^2\theta - 1)$
3	2	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{15}}{2} \sin\theta\cos\theta$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin\theta\cos\theta\, e^{\pm i\phi}$
3	2	± 2	$\frac{1}{\sqrt{2\pi}} e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4} \sin^2\theta$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin^2\theta\, e^{\pm 2i\phi}$

*The quantity $a_0 = 4\pi\epsilon_0\hbar^2/me^2 = 5.292 \times 10^{-11}$ m is equal to the radius of the innermost Bohr orbit.

Figure 1: Normalised wave functions of the hydrogen atom for $n = 1, 2$ and 3.