University of the Witwatersrand
Physics IIE (Engineering) : PHYS284 - 2005
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Quantum Mechanics, Statistical Mechanics, Semiconductivity

Class Test 2 — 60 + 10 min. (60 marks)

1. a) Consider the Schrödinger Wave Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + U(x)\psi(x) = E\psi(x)$$

for a particle interacting with the potential

$$U(x) = \begin{cases} 0 & x < 0 & \text{region I} \\ U & 0 \leq x \leq L & \text{region II} \\ 0 & x > L & \text{region III} \end{cases} .$$

Sketch the situation, labelling the sketch fully. (4)

b) For a particle incident from the left, show that the solution in the barrier region is not the normal oscillatory wave function, but has a decreasing exponential form.

$$\psi = Be^{-\frac{\sqrt{2m(U-E)}}{\hbar}x}$$

(5)

c) The fact that the wave function penetrates the barrier leads to the quantum phenomenon of tunneling. The probability for this is approximately

$$T = e^{-2\frac{\sqrt{2m(U-E)}}{\hbar}L}.$$

i) Indicate the meaning of the symbols $(U-E)$, L and m. (3)

ii) In the corresponding classical process, the particle has to hop over the barrier with probability

$$P = e^{-\frac{(U-E)}{kT}}.$$

The tunneling process represents a physics limit for the miniaturisation of features on a chip. This could be either tunneling between neighbouring wires or across the gate of a transistor. Imagine that the quantum process should not be more likely than the classical process, so that the limiting case is when they are equal. This will allow you to estimate a value for the minimum feature size L of a conventional chip. Estimate realistic values for $(U-E)$ and kT and explain your choice. (2)

iii) Perform the calculation to find L. (4)

[18]

2. (a) The probability of finding the electron in the hydrogen atom ground state, at some distance between r and $r + dr$ is given by

$$P(r)dr = \frac{1}{a_0}\left(\frac{2r}{a_0}\right)^2 e^{-2r/a_0}\,dr$$

where a_0 is the Bohr radius (0.05292 nm). Make a rough plot of $P(r)$ in terms of r/a_0 and calculate the distance from the nucleus where the electron is most likely to be found. (10)

(b) The energy levels of certain kinds of two-electron atoms/ions of atomic number Z may be approximated by

$$E_n(Z) = E_1(H)Z^2 - \frac{E_1(H)(Z-1)^2}{n^2}$$

where the ground state of hydrogen is $E_1(H) = -13.6\text{eV}$.

i) Sketch the two-electron atom/ion under conditions where the above expression could be expected to hold. (4)

ii) Using the full expression for quantised energy levels in hydrogen-like (that is, one-electron) atoms/ions of atomic number Z, describe how each term in the above expression arises. (4)

iii) Why does the accuracy increase when n increases ? (2)

[20]

3. (a) Evaluate the Fermi-Dirac distribution function for $T = 0\,\text{K}$ below and above the Fermi energy, ϵ_F, and plot the result. (4)

(b) Describe how this result is appropriately consistent with Pauli's exclusion principle, and discuss which electrons can participate in transport processes. (4)

[8]

4. The number of electrons $n(\epsilon)d\epsilon$ with energies from ϵ to $\epsilon + d\epsilon$ in a Fermi gas is

$$\begin{aligned} n(\epsilon)d\epsilon &= g(\epsilon)f_{FD}(\epsilon)d\epsilon \\ &= \frac{8\sqrt{2}\pi V m^{3/2}}{h^3}\frac{\sqrt{\epsilon}d\epsilon}{e^{(\epsilon-\epsilon_F)/kT}+1} \end{aligned}$$

Calculate the number of charge carriers thermally excited into the conduction band in a semiconductor.

(a) Start by presenting a labelled block sketch of the band structure for a pure semiconductor, indicating the valence band, conduction band and the labelled energy axis. Indicate the presence of electrons at low temperature by appropriate shading. (4)

(b) Three modifications from the Fermi gas scenario to the semi-conductor scenario are required. These are given below. Write a justification explaining each of them.

$$\frac{8\sqrt{2}\pi V m^{3/2}}{h^3}\sqrt{\epsilon} \longrightarrow \frac{8\sqrt{2}\pi V m^{3/2}}{h^3}\sqrt{\epsilon-\epsilon_g}$$
$$\epsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{3/2} \longrightarrow \frac{h^2}{2m^*}\left(\frac{3N}{8\pi V}\right)^{3/2}$$
$$\frac{1}{e^{(\epsilon-\epsilon_F)/kT}-1} \longrightarrow e^{-(\epsilon-\epsilon_F)/kT}.$$

(3)

(c) Perform the calculation for the number of electrons in the conduction band of an intrinsic semi-conductor at finite temperature and show that

$$n_e = \int_{\epsilon_g}^{\infty} n(\epsilon)\, d\epsilon = 2V\left(\frac{m^* kT}{2\pi\hbar^2}\right)^{3/2} e^{(\epsilon_F-\epsilon_g)/kT}.$$

(Hint $\int_0^\infty x^{1/2}e^{-x}\,dx = \Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2}$) (5)

(d) The band gap of pure silicon is 1.12 eV, and the effective mass of the electron is $m_e^* = 0.2m_e$. Calculate the density of conduction electrons at room temperature. Take $\epsilon_F = \epsilon_g/2$. (2)

[14]

Total Marks [60]