University of the Witwatersrand
Physics IIE (Engineering) : PHYS284 : 2001
Examination : June 2001

Instructions: Answer all questions.
Time: 2 hours = 120 minutes
Total Marks: 120 marks

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1. (a) A proton in an accelerator is moving at a speed of $0.9c$ as seen from the laboratory. An electron must be accelerated to overtake the the proton with a speed relative to the proton of $0.5c$ in the same direction, what must be the speed of the electron in the laboratory -
   i) calculated using Galilean Relativity ? (3)
   ii) calculated using Special Relativity ? (7)

   (b) What approximate fractional error does one make in using
   i) $\frac{1}{2}m_0v^2$ and
   ii) $\frac{1}{2}mv^2$

   for the kinetic energy of a body if its speed $v = 0.1c$, and again if its speed $v = 0.9c$? Give fractions to two significant figures.
   (Hint : fractional error = (true value - approximate value)/true value).) (10)

   [20]

2. A linear quantum dot can be modeled as an electron in a box. It can be formed by depositing contacts on a slab of GaAs and then applying a bias voltage in such a way as to create a linear confining potential.
   (Note : for GaAs, $m^* = 0.067m_e$.)

   (a) Suppose the observed ground-state electron energy is 0.003 eV. How big is the potential boundary ("box") of the quantum dot ? (8)

   (b) Find the expectation value x of the position of an electron in this dot as a function of its quantum number n.
   (Reminder: $\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$ and $P(x)dx = |\psi(x)|^2dx$ with $\langle x\rangle = \int xP(x)\,dx$.)
   (You may need : $2\sin^2 t = 1 - \cos 2t$.) (12)

   [20]

3. (a) The probability of finding the electron in the hydrogen atom ground state, at some distance between r and $r + dr$ is given by

$$P(r)dr = \frac{1}{a_0}\left(\frac{2r}{a_0}\right)^2 e^{-2r/a_0}\,dr$$

   where a_0 is the Bohr radius (0.05292 nm). Make a rough plot of $P(r)$ in terms of r/a_0 and calculate the distance from the nucleus where the electron is most likely to be found. (15)

(b) The energy levels of certain kinds of two-electron atoms/ions of atomic number Z may be approximated by

$$E_n(Z) = E_1(H)Z^2 - \frac{E_1(H)(Z-1)^2}{n^2}$$

where the ground state of hydrogen is $E_1(H) = -13.6\text{eV}$.

i) Sketch the two-electron atom/ion under conditions where the above expression could be expected to hold. (5)

ii) Using the full expression for quantised energy levels in hydrogen-like (that is, one-electron) atoms/ions of atomic number Z, describe how each term in the above expression arises. (6)

iii) Why does the accuracy increase when n increases ? (4)

[30]

4. (a) Starting with the wave-functions for two identical non-interacting quantum particles, show that the probability for two fermions to occupy the same quantum state is zero, and that the probability for two bosons to occupy the same quantum state is twice that for two classical particles to occupy the same state.
(Note : $\Psi_{\frac{B}{F}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}}[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) \pm \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)]$.) (6)

(b) Use this result to state and explain the relative pressure exerted by similar gases of classical molecules, bosons, or fermions for the same temperature. (4)

(c) Sketch the Fermi-Dirac distribution (see question 5a) at low but finite temperature and explain why only electrons near the Fermi-level are expected to participate in transport properties. (4)

(d) Show for this distribution (see question 5a) that if the average occupancy of a state of energy $\epsilon_F + \Delta\epsilon$ is f_1 at any temperature, then the average occupancy of a state of energy $\epsilon_F - \Delta\epsilon$ is $1 - f_1$.

(6)

[20]

5. (a) In an intrinsic semiconductor, $n(\epsilon)d\epsilon = g(\epsilon)f_{FD}(\epsilon)d\epsilon$, where

$$f_{FD}(\epsilon) = \frac{1}{e^{(\epsilon-\epsilon_F)/kT} + 1}$$

and

$$g_e(\epsilon)d\epsilon = \frac{8\sqrt{2}\pi V m_e^{*3/2}}{h^3}\sqrt{\epsilon - \epsilon_g}\, d\epsilon, \qquad g_h(\epsilon)d\epsilon = \frac{8\sqrt{2}\pi V m_h^{*3/2}}{h^3}\sqrt{-\epsilon}\, d\epsilon.$$

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Show that the number of electrons in the conduction band is :

$$n_e = 2V \left(\frac{m_e^* kT}{2\pi\hbar^2} \right)^{3/2} e^{(\epsilon_F - \epsilon_g)/kT}.$$

(Hint : $\int y^{\frac{1}{2}} e^{-y}\, dy = \frac{\pi^{\frac{1}{2}}}{2}$.) (7)

(b) A similar equation can be shown to hold for the number of holes in the valence band :

$$n_h = 2V \left(\frac{m_h^* kT}{2\pi\hbar^2} \right)^{3/2} e^{-\epsilon_F/kT}.$$

Considering this, show

$$\epsilon_F = \epsilon_g/2$$

at room temperature in an intrinsic semiconductor. (6)

(c) Also derive the expression

$$n_i = n_0 e^{-\epsilon_g/2kT}$$

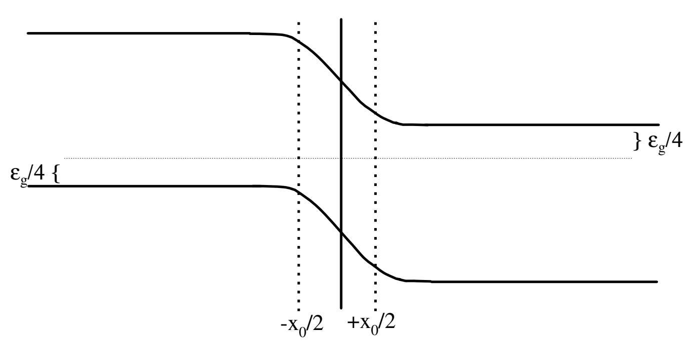
for the number of intrinsic carriers and give the explicit form of n_0. (6)

(d) Explain the term "donor" (include a sketch). In this case, the number of carriers is

$$n_c = n_i e^{(\epsilon_F - \epsilon_i)/kT}.$$

(6)

(e) A semiconductor is characterised by the energy band structure shown in the following figure. Determine the resistivity of the $x_0 > x_0/2$ section of the semiconductor. ($\epsilon_g = 1.12\text{eV}$, $n_i = 1.18 \times 10^{10}\text{cm}^{-3}$, $kT = 0.026\text{eV}$ $\mu = 1350\text{cm}^2\text{V}^{-1}\text{s}^{-1}$, $e = 1.6 \times 10^{-19}\text{C}$.) (5)



[30]

Total Marks **[120]**