University of the Witwatersrand
Physics IIE (Engineering) : PHYS284 : 2004
Examination : June 2004

Instructions: Answer all questions.
Time: 2 hours = 120 minutes
Total Marks: 120 marks

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1. a) Starting from

$$W = E_k = \int_0^s F ds = \int_0^s \frac{d(mv)}{dt} ds$$

Show that the relativistic expression for the kinetic energy is

$$E_k = mc^2 - m_0c^2.$$

(8)

b) State the physical meaning of the two terms on the right hand side of the previous expression. (2)

c) Show that the non-relativistic expression can be recovered at low velocities. (5)

d) The Global Positioning System (GPS) consists of satellites with orbital speeds of about 3.9 km/s in a frame centred on the Earth. The orbital radius of the satellites is about 26,600 km.

i) Do the satelite bound clocks tick faster or slower than the earth bound clocks, considering the effects of Special Relativity ? (1)

ii) If 12 hours have passed on the satellite, what would the elapsed time have been on the earth, considering the effects of Special Relativity ? Express your answer as a time difference in microseconds.
(Use an approximate method if your calculator has insufficient accuracy.) (3)

iii) What position error would this represent, if the corrections were not made ? (3)

iv) What procedure is implemented to avoid this error ? (3)

Total for Question 1 [25]

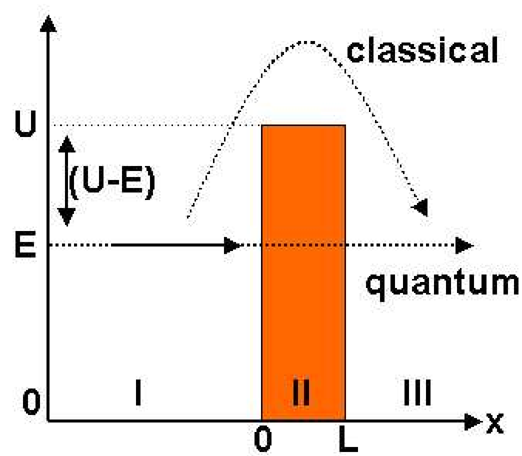
2. a) Consider the Schrödinger Wave Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + U(x)\psi(x) = E\psi(x)$$

with the potential

$$U(x) = \begin{cases} 0 & x < 0 & \text{region I} \\ U & 0 \le x \le L & \text{region II} \\ 0 & x > L & \text{region III} \end{cases},$$

as in the figure.



For a particle incident from the left, show that the solution in the barrier region is not the normal oscillatory wave function, but has a decreasing exponential form.

$$\psi = Be^{-\frac{\sqrt{2m(U-E)}}{\hbar}x}$$

(6)

b) The fact that the wave function penetrates the barrier leads to the quantum phenomenon of tunneling. The probability for this is approximately

$$T = e^{-2\frac{\sqrt{2m(U-E)}}{\hbar}L}.$$

i) Indicate the meaning of the symbols $(U - E)$, L and m. (4)

ii) In the corresponding classical process, the particle has to hop over the barrier with probability

$$P = e^{-\frac{(U-E)}{kT}}.$$

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The tunneling process represents a physics limit for the miniaturisation of features on a chip. This could be either tunneling between neighbouring wires or across the gate of a transistor. Imagine that the quantum process should not be more likely than the classical process, so that the limiting case is when they are equal. This will allow you to estimate a value for the minimum feature size L of a conventional chip. Estimate realistic values for $(U - E)$ and kT and explain your choice. (4)

iii) Perform the calculation to find L. (6)

Total for Question 2 [20]

3. (a) The probability of finding the electron in the hydrogen atom ground state, at some distance between r and $r + dr$ is given by

$$P(r)dr = \frac{1}{a_0}\left(\frac{2r}{a_0}\right)^2 e^{-2r/a_0}\, dr$$

where a_0 is the Bohr radius (0.05292 nm). Make a rough plot of $P(r)$ in terms of r/a_0 and calculate the distance from the nucleus where the electron is most likely to be found. (15)

(b) The energy levels of certain kinds of two-electron atoms/ions of atomic number Z may be approximated by

$$E_n(Z) = E_1(H)Z^2 - \frac{E_1(H)(Z-1)^2}{n^2}$$

where the ground state of hydrogen is $E_1(H) = -13.6\text{eV}$.

i) Sketch the two-electron atom/ion under conditions where the above expression could be expected to hold. (5)

ii) Using the full expression for quantised energy levels in hydrogen-like (that is, one-electron) atoms/ions of atomic number Z, describe how each term in the above expression arises. (6)

iii) Why does the accuracy increase when n increases ? (4)

Total for Question 3 [25]

4. (a) A cubic meter of atomic hydrogen at 0°C and at atmospheric pressure contains about 2.7×10^{25} atoms. Find the ratio of the number of these atoms in their first excited state ($n = 2$) to the number in the ground state ($n = 1$) at 10,000°C. (6)

(b) Show that in a system of fermions at $T = 0\,\mathrm{K}$, all states with $\epsilon < \epsilon_F$ are occupied, while all those with $\epsilon > \epsilon_F$ are unoccupied. (4)

(c) The number of fermions in a fermi gas (for example, electrons in a metal) that have energies from ϵ to $\epsilon + d\epsilon$ is

$$\begin{aligned} n(\epsilon)d\epsilon &= g(\epsilon)f_{FD}(\epsilon)d\epsilon \\ &= \frac{8\sqrt{2}\pi L^3 m^{3/2}}{h^3}\frac{\sqrt{\epsilon}d\epsilon}{e^{(\epsilon-\epsilon_F)/kT}+1} \end{aligned}$$

Consider N electrons in the low temperature limit where all the low-lying states are occupied. Show that the energy of the last filled state, known as the fermi energy, is given by

$$\epsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}.$$

(7)

(d) The density of metallic zinc is 7.13 g/cm^3 and the atomic mass of the zinc atom is 65.4 u. The Fermi energy in zinc metal is 11.0 eV.

(i) Work out the effective mass of a delocalised electron in zinc metal. Express your answer in terms of the free electron mass.
(Zinc has 2 valence electrons.) (6)

(ii) Why is there a difference ? (2)

Total for Question 4 [25]

5. (a) In a one-dimensional lattice, the energy of an electron in the valence band as a function of its wave number, k, may be approximated by,

$$E(k) = E_0(a^2k^2 - \frac{a^4}{2\pi^2}k^4)$$

where E_0 is a positive constant, and a is the lattice spacing. What is the effective mass an electron at the top of the band $(k = \pi/a)$?
(Express your answer in terms of E_0, a and $\hbar$). (6)

(b) Derive the distribution of holes in an intrinsic semiconductor from the statement that $f_h(\epsilon) = 1 - f_{FD}(\epsilon)$. (6)

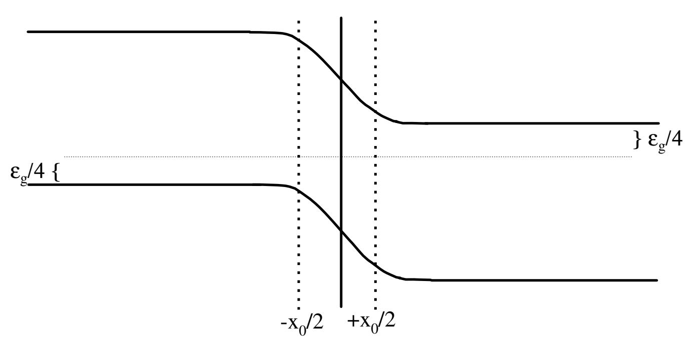
(c) A semiconductor is characterised by the energy band structure shown in the following figure.

(i) Specify which side of the diagram has acceptor dopants, and which side has electrons as charge carriers. (2)

(ii) What is the concentration of charge carriers in the $x > x_0/2$ section of the semiconductor ? (6)

(iii) What is the resistivity in this section ?

(5)



$(\epsilon_g = 1.12\text{eV},\ n_i = 1.18 \times 10^{10}\text{cm}^{-3},\ kT = 0.026\text{eV}\ \mu = 1350\text{cm}^2\text{V}^{-1}\text{s}^{-1},\ e = 1.6 \times 10^{-19}\text{C}.)$

Total for Question 6 [25]

Total Marks **[120]**

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