Formula Sheet

$$N_A = 6.022 \times 10^{23}, \quad k = 1.381 \times 10^{-23}\text{J/K} = 8.617 \times 10^{-5}\text{eV/K}, \quad c = 2.997 \times 10^{8}\text{m/s}$$

$$h = 6.626 \times 10^{-34}\text{J.s}, \quad \hbar = 1.055 \times 10^{-34}\text{J.s}, \quad R = 8.314 \times 10^{3}\text{J/kmol}, \quad m_e = 9.109 \times 10^{-31}\text{kg}$$

$$u = 1.66 \times 10^{-27}\text{kg}, \quad \kappa = 2.898 \times 10^{-3}\text{m.K}, \quad e = 1.609 \times 10^{-19}\text{C}, \quad \epsilon_0 = 8.854 \times 10^{-12}\text{C}^2/\text{N.m}^2$$

$$\mu_0 = 4\pi \times 10^{-7}\text{T.m/A}, \quad a_0 = 0.529 \times 10^{-10}\text{m}, \quad E_1(H) = -13.61eV$$

$$\begin{array}{rcl} x' &=& x - vt \\ y' &=& y \\ z' &=& z \\ t' &=& t \end{array}, \quad \begin{array}{rcl} x' &=& \frac{(x-vt)}{\sqrt{1-v^2/c^2}} \\ y' &=& y \\ z' &=& z \\ t' &=& \frac{(t-\frac{vx}{c^2})}{\sqrt{1-v^2/c^2}} \end{array}, \quad \begin{array}{rcl} v'_x &=& \frac{v_x - v}{1-\frac{vv_x}{c^2}} \\ v'_y &=& \frac{v_y\sqrt{1-v^2/c^2}}{1-\frac{vv_x}{c^2}} \\ v'_z &=& \frac{v_z\sqrt{1-v^2/c^2}}{1-\frac{vv_x}{c^2}}. \end{array}, \quad \begin{array}{rcl} \beta &=& v/c \\ \gamma &=& \frac{1}{\sqrt{1-v^2/c^2}} \\ L = L_0/\gamma &,& t = \gamma t_0 \end{array}$$

$$\nu = \nu_0\left(\frac{1+v/c}{1-V/c}\right), \quad \nu_\perp = \nu_0/\gamma, \quad \nu_\downarrow = \nu_0\left(\frac{1-v/c}{1+v/c}\right)^{1/2}, \quad \nu_\uparrow = \nu_0\left(\frac{1+v/c}{1-v/c}\right)^{1/2}.$$

$$m = m_0/\sqrt{1-v^2/c^2} = \gamma m_0, \quad p = mv = \gamma m_0 v = \frac{m_0 v}{\sqrt{1-v^2/c^2}}, \quad F = \frac{d}{dt}(mv)$$

$$E_k = mc^2 - m_0c^2, \quad E = mc^2, \quad E_0 = m_oc^2, \quad E = E_0 + E_k$$

$$E^2 = m_0^2c^4 + p^2c^2, \quad E = pc,$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi(x,y,z) + U(x,y,z)\psi(x,y,z) = E\psi(x,y,z)$$

$$\Psi(x,y,z,t) = \psi(x,y,z)e^{-(iE/\hbar)t}, \quad P(\boldsymbol{r}) = |\psi(\boldsymbol{r})|^2\, d^3\boldsymbol{r}\ , \quad \int_{-\infty}^{+\infty} |\psi(\boldsymbol{r})|^2\, d^3\boldsymbol{r}\ = 1.$$

$$\Delta p \Delta x \approx \hbar/2, \quad \langle G(\boldsymbol{r})\rangle = \int_{-\infty}^{+\infty} G(\boldsymbol{r})|\psi(\boldsymbol{r})|^2\, d^3\boldsymbol{r}\ , \quad \Psi(\boldsymbol{r},t) = Ae^{i(\boldsymbol{k}.\boldsymbol{r} - Et/\hbar)}.$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \quad \text{n=1,2,3...}, \quad \psi_n(x) = \sqrt{\frac{2}{L}}\sin k_n x = \sqrt{\frac{2}{L}}\sin\frac{\sqrt{2mE_n}}{\hbar}x = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}.$$

$$T \approx e^{-2k_2L}, \quad k_2 = \frac{\sqrt{2m(U-E)}}{\hbar}.$$

$$m_r = m_p m_e/(m_p + m_e), \quad U(x,y,x) = U(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}, \quad dv = dx\,dy\,dz = r^2\sin\theta\,dr\,d\theta\,d\phi = d^3r$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

$$\frac{d^2\Phi}{d\phi^2} + m_l^2\Phi = 0, \quad \frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \left[\frac{Ze^2}{4\pi\epsilon_0}\frac{1}{r} - \frac{l(l+1)\hbar^2}{2mr^2} + E\right]R = 0$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + \left[l(l+1) - \frac{m_l^2}{\sin^2\theta}\right]\Theta = 0, \quad \Phi_{m_l}(\phi) = Ae^{im_l\phi}$$

$$\psi_{nlm_l}(\mathbf{r}) = R_{nl}(r)\Theta_{lm_l}(\theta)\Phi_{m_l}(\phi), \quad E_n = -\frac{mZ^2e^4}{32\pi^2\epsilon_0^2\hbar^2}\left(\frac{1}{n^2}\right) = \frac{E_1}{n^2}, \quad n = 1,2,3,\cdots$$

$$n = 1,2,3,\cdots, \quad l = 0,1,2,3,\cdots,(n-1), \quad m_l = 0,\pm1,\pm2,\pm3,\cdots,\pm l \quad m_s = \pm\tfrac{1}{2}$$

$$L^2 = \hbar^2 l(l+1), \quad L_z = m_l\hbar, \quad M_L = -\frac{e\hbar}{2m_e}m_l, \quad E_{\text{mag}} = -M_L.B = -\frac{e\hbar}{2m_e}m_lB$$

$$S^2 = \hbar^2 s(s+1), \quad S_z = m_s\hbar, \quad M_S = -\frac{2e\hbar}{2m_e}m_s, \quad P_{n,l}(r)dr = r^2|R_{n,l}(r)|^2dr$$

$$E_{\text{photon}} = h\nu = \hbar\omega = E_{n'} - E_n, \quad \Delta l = \pm1, \quad \Delta m_l = 0,\pm1$$

$$|\Psi(\mathbf{r}_1,\mathbf{r}_2)|^2 = |\Psi(\mathbf{r}_2,\mathbf{r}_1)|^2, \quad \Psi_I(\mathbf{r}_1,\mathbf{r}_2) = \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2), \quad \Psi_{II}(\mathbf{r}_1,\mathbf{r}_2) = \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)$$

$$\Psi(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\sqrt{2}}[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)+\psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)],\qquad \Psi(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\sqrt{2}}[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)-\psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)]$$

$$f_{MB}(\epsilon)=Ae^{-\epsilon/kT},\qquad n(\epsilon)d\epsilon=g(\epsilon)f_{MB}(\epsilon)d\epsilon=\frac{2\pi N}{(\pi kT)^{3/2}}\sqrt{\epsilon}\,e^{-\epsilon/kT}d\epsilon,\qquad \overline{\epsilon}=\tfrac{3}{2}kT$$

$$f_{BE}(\epsilon)=\frac{1}{e^{\,\epsilon/kT}-1},\qquad n(\epsilon)d\epsilon=g(\epsilon)f_{BE}(\epsilon)d\epsilon=\frac{8\pi\nu^2}{c^3}\frac{1}{e^{\epsilon/kT}-1}d\nu$$

$$u(\nu,T)\,d\nu=\frac{8\pi h}{c^3}\frac{\nu^3\,d\nu}{e^{h\nu/kT}-1}.,\qquad \lambda_{\text{max}}T=\kappa,\qquad R=e\sigma T^4,\qquad j^2=(j_x^2+j_y^2+j_z^2)=\left(\frac{2L}{\lambda}\right)^2,$$

$$f_{FD}(\epsilon)=\frac{1}{e^{(\,\epsilon-\epsilon_F)/kT}+1},\qquad n(\epsilon)d\epsilon=g(\epsilon)f_{FD}(\epsilon)d\epsilon=\frac{8\sqrt{2}\pi L^3m^{3/2}}{h^3}\frac{\sqrt{\epsilon}d\epsilon}{e^{(\,\epsilon-\epsilon_F)/kT}+1}$$

$$\lambda=h/p,\qquad \epsilon_F=\frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3},\qquad \overline{\epsilon}=\tfrac{3}{5}\epsilon_F.$$

$$U=-\frac{\alpha e^2}{4\pi\epsilon_0 r_0}\cdot\left(1-\frac{1}{n}\right),\qquad E_C=\tfrac{1}{2}(E_I+E_A+U)$$

$$\tau=\lambda/v_F,\qquad v_d=\frac{V}{L}\frac{e}{m}\cdot\frac{\lambda}{v_F},\qquad V=IR,\qquad R=\left(\frac{mv_F}{ne^2\lambda}\right)\cdot\frac{A}{L}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)+U(x)\psi(x)=E\psi(x),\quad \psi(x)=u_k(x)e^{-kx},\quad u_k(x+L)=u_k(x)$$

$$\alpha^2=\frac{2mE}{\hbar^2},\quad \beta^2=\frac{2m(V_0-E)}{\hbar^2}$$

$$u_{k_1}(x)=A\exp\left[i(\alpha-k_1)x\right]+B\exp\left[-i(\alpha+k_1)x\right],\quad u_{k_2}(x)=C\exp\left[i(\beta-k_2)x\right]+D\exp\left[-i(\beta+k_2)x\right]$$

$$K\frac{\sin(\alpha a)}{\alpha a}+\cos(\alpha a)=cos(ka),\quad K=\frac{mV_0ba}{\hbar^2},\quad -1<K\frac{\sin(\alpha a)}{\alpha a}+\cos(\alpha a)<1$$

$$n\lambda=2a\sin\theta,\quad k\sin\theta=\frac{n\pi}{a}=k_x,\quad \psi\propto\cos\left(\frac{n\pi x}{a}\right),\quad \psi\propto\sin\left(\frac{n\pi x}{a}\right).$$

$$v_p=\frac{\omega}{k},\quad v_g=\frac{d\omega}{dk}=\frac{1}{\hbar}\frac{dE}{dk},\quad F=m^*\frac{dv}{dt}=\frac{dp}{dt}=\hbar\frac{dk}{dt}.$$

$$a=\frac{dv}{dt}=\frac{dv}{dk}\frac{dk}{dt},\quad a=\frac{d}{dk}\left(\frac{1}{\hbar}\frac{dE}{dk}\right)\frac{dk}{dt},\quad m^*=\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}.$$

$$\epsilon_F=\frac{h^2}{2m^*}\left(\frac{3N}{8\pi V}\right)^{2/3},\quad f_h(\epsilon)=1-f_{FD}(\epsilon)\qquad g(\epsilon)d\epsilon=\frac{8\sqrt{2}\pi Vm_e^{*3/2}}{h^3}\sqrt{\epsilon-\epsilon_g},\quad g(\epsilon)d\epsilon=\frac{8\sqrt{2}\pi Vm_h^{*3/2}}{h^3}\sqrt{-\epsilon}$$

$$n(\epsilon)d\epsilon=g(\epsilon)f_{FD}(\epsilon)d\epsilon,\quad n_e=2V\left(\frac{m_e^*kT}{2\pi\hbar^2}\right)^{3/2}e^{(\epsilon_F-\epsilon_g)/kT},\quad n_h=2V\left(\frac{m_h^*kT}{2\pi\hbar^2}\right)^{3/2}e^{-\epsilon_F/kT}.$$

$$\epsilon_F=\frac{\epsilon_g}{2}+\tfrac{3}{4}kT\ln\left(\frac{m_h^*}{m_e^*}\right),\quad n_e=n_h=n_i=2V\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2}(m_e^*m_h^*)^{3/4}e^{-\epsilon_g/2kT}$$

$$E_n=-\frac{mZ^2e^4}{32\pi^2(\epsilon_0\epsilon_r)^2\hbar^2}\left(\frac{1}{n^2}\right)=\frac{E_1}{n^2},\quad n=1,2,3,\cdots,\quad \frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2m^*r^2}{\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0\epsilon_r r}+E\right)=0$$

$$\epsilon_F=\frac{\epsilon_d+\epsilon_g}{2}+\frac{kT}{2}\ln\left[\frac{N_d}{2N_c}\right],\quad \epsilon_F=\frac{\epsilon_a+0}{2}-\frac{kT}{2}\ln\left[\frac{N_a}{2N_v}\right],\quad \epsilon_i=\frac{\epsilon_g}{2}$$

$$n_np_n=n_i^2,\quad p_pn_p=n_i^2,\quad n_n=N_d-N_a,\quad p_n=\frac{n_i^2}{N_d-N_a},\quad p_p=N_a-N_d,\quad n_p=\frac{n_i^2}{N_a-N_d}.$$

$$J_f=eD\frac{dn_v}{dx},\quad v_d=\mu\xi,\quad J_d=v_dn_ve=\mu n_ve\xi,\quad \rho=\xi/J$$

$$n_v=n_{v0}e^{-\frac{nu}{D}\phi},\quad n_v=n_{v0}e^{-\frac{e\phi}{kT}},\quad \frac{\mu_n}{D_n}=\frac{e}{kT},\quad \frac{\mu_p}{D_p}=\frac{e}{kT},\quad D=\frac{\mu kT}{e}.$$

$$\xi=\frac{1}{e}\frac{d\epsilon_i}{dx},\quad F_p=D_p\frac{p_n}{kT}\frac{d\epsilon_F}{dx},\quad \frac{d\epsilon_F}{dx}=0.$$

$$T = \frac{2nL}{c} = \frac{1}{\Delta v_g}, \qquad \tau = \frac{2nL}{Nc} = \frac{1}{N\Delta v_g}, \qquad < P >= Ef, \qquad P_p = E/\tau$$

$$I_{nm}(x,y,z) = I_0 \left[H_m \left(\frac{x\sqrt{2}}{w(z)} \right) \exp \left(\frac{-x^2}{w^2(z)} \right) H_n \left(\frac{y\sqrt{2}}{w(z)} \right) \exp \left(\frac{-y^2}{w^2(z)} \right) \right]^2, \qquad I(x,y,z) = I_0 g(t) f(x,y,z)$$

$$\omega(z) = \omega_0 \left[1 + \left(\frac{M^2 \lambda (z - z_0)}{\pi \omega_0^2} \right)^2 \right], \qquad \theta = \omega(z)/z, \qquad \omega_0 \theta_0 = \frac{M^2 \lambda}{\pi}$$

$$z_R = \frac{\pi \omega_0^2}{M^2 \lambda}, \qquad R(z) = (z - z_0) \left[1 + \left(\frac{\pi \omega_0^2}{M^2 \lambda} \right)^2 \right]$$

$$\omega^2(z) = \left(1 - \frac{z}{f} \right)^2 \omega_0^2 + \left(z + \left(1 - \frac{z}{f} \right) d \right)^2 \theta_0^2, \qquad \theta^2 = \left(\frac{-1}{f} \right)^2 \omega_0^2 + \left(1 - \frac{d}{f} \right)^2 \theta_0^2, \qquad \theta_0 = \omega_f / f$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, \qquad z = \frac{f(\omega_0^2 + \theta_0^2 (d - f) d)}{\omega_0^2 + \theta_0^2 (d - f)^2}$$