University of the Witwatersrand
Physics IIE (Engineering) : PHYS284 - 2004

Tutorial 1 : Relativistic Mechanics

1. Obtain the relativistic velocity transformation, hence show that the speed of light is the same in all inertial reference frames.
(Hint: write down the Lorentz transformation for infinitesimal displacements and times)

2. A sample of radioactive material, at rest in the laboratory, ejects two electrons in opposite directions with speeds of $0.6c$ and $0.7c$, as measured by a laboratory observer. What is the speed of one electron as measured from the other, according to

   i) classical physics ?

   ii) Einstein's relativity ?

   [$1.3c$ and $0.92c$]

3. A muon is created by cosmic ray activity in the upper atmosphere travelling to earth at 99.5% of the speed of light. It decays after two microseconds in its own rest-frame, just as it reaches the earth's surface.

   i) Evaluate the Lorentz factor.

   ii) In the muon-frame, how far did it travel ?

   iii) In the muon-frame, how far did the earth travel ?

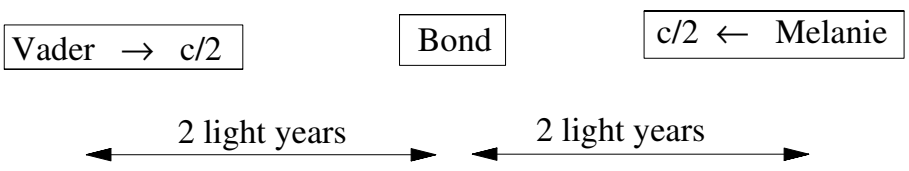
   iv) In the earth-frame, for how long did the muon travel ?

   v) In the earth-frame, how far did the muon travel ?

   [10.01, 0 597m, 20μs, 5.97km]

4. A neutron is created in the upper atmosphere of the sun travelling towards the earth's orbit. It decays after 1000 seconds (in its rest frame) just as it reaches the orbit. How fast was it travelling ?
(In the rest frame of the sun, the earth is 150 Gm away.) [0.447c]

5. James Bond is blindfolded and strapped to a meteorite. The law-enforcement radar micro-chip implanted in his skull tells him two rockets. initially 2 light years away, are approaching him at half the speed of light from opposite directions.



   i) How long does Bond think he has before the "bad" rocket ship (driven by Darth Vader) reaches him ?

ii) How long does princess Melanie (driving the "good" rocket ship) think she will take to reach Bond ?

iii) At what speed do her sensors tell her Darth Vader is approaching her ?

[4 years, 3.464 years, $\frac{4}{5}c$]

6. The energy-momentum relationship is

$$E^2 = (K + M_0c^2)^2 = P^2c^2 + M_0^2c^4 = M^2c^4.$$

(a) Show that it can be written as $e^2 = (k+1)^2 = p^2 + 1$, also that $k^2 + 2k = p^2$ where $e = E/M_0c^2, k = K/M_0c^2$ and $p = P/M_0c$.

(b) Show that $k + 1 = \gamma = e$ and $p = \beta\gamma$.

(c) Show that the velocity-momentum relationship can be written $1 + 1/p^2 = 1/\beta^2$.

(d) Show that the de Broglie relationship ($\lambda_D = h/p$) can be written as $p = \lambda_C/\lambda_D$ where $\lambda_C = h/M_0c$, the Compton wavelength.

7. (a) Evaluate γ for speeds 20%, 40%, 60%, 60%, 90% and 95% of the speed of light. Draw graphs of γ as a function of β.

(b) What approximate fractional error does one make in using

i) $\frac{1}{2}m_0u^2$ and

ii) $\frac{1}{2}mu^2$

for the kinetic energy of a body if its speed u is 3 m/s, 30 km/s, 30 Mm/s, and 270 Mm/s. Give fractions to two significant figures (fractional error = (true value - approximate value)/true value).

(i) $[0.75 \times 10^{-16}, 0.75 \times 10^{-8}, 0.0075, 0.69]$
(ii) $[0.25 \times 10^{-16}, 0.25 \times 10^{-8}, 0.0025, 0.28]$

8. Electrons are accelerated from rest to a speed v by passing through a potential difference V. Calculate V if

i) $v = 5c/13$

ii) $v = 0.99$c

(take $m_e = 9.11 \times 10^{-31}$ kg, $m_ec^2 = 511$ keV)

[42.6 kV, 3.11 MV]

9. Compute the daily loss of mass of the sun associated with its emission of electromagnetic radiation given that the average solar constant at the earth is 1400 W/m^2 and that the mean radius of the earth's orbit is 150 Gm.

[3.8×10^{14} kg]

10. A beam of 5×10^{15} electrons per second (velocity $0.99c$) is being stopped in a 1 kg block of lead.

i) What is the average force exerted on the block by the beam ?

ii) If the initial temperature of the block is 20°C, how long would it take to raise its temperature to its melting point of 327°C, assuming the specific heat of lead is a constant 160 J.kg^{-1}°C^{-1}, and that there are no heat losses from the block.

[19.74s]

11. Is it possible for an electron beam in a television tube to move across the screen at a speed faster than the speed of light ? Why does this not contradict special relativity ?

12. The Global Positioning System (GPS) consists of a network of 24 satellites in roughly 12-hour orbits, each carrying atomic clocks on board. The orbital radius of the satellites is about 26,600 km. The satellites have orbital speeds of about 3.9 km/s in a frame centred on the Earth. The on-board atomic clocks have a period of about 1 nanosecond (ns) and they have a rate accuracy of 1 ns/day. The GPS receiver determines its current position and heading by comparing the time signals it receives via a radio transmission from a number of the GPS satellites (usually 4 to 12) and triangulating on the known positions of each satellite.

    i) Consider the distance that light can travel in 1 ns and determine the accuracy with which the timing information from the radio transmisions must be known in order to correspond to the typical position accuracy of 5m.

    The timing signals of the clocks are affected by both General and Special Relativity. (The effects of General Relativity are about nine times larger than the those of Special Relativity, but they are not considered here.)

    ii) Considering the Special Relativity effects, will the satellite based clock tick slower or faster than the ground based clock as seen from the ground based clock ? Elaborate on your answer briefly.

    iii) Now show that the time difference between the satellite based clock, and the ground based clock, amounts to about 7 μs per day.