University of the Witwatersrand

Physics IIE (Engineering) : PHYS284 - 2004

Tutorial 2 : Introduction to Quantum Mechanics

1. Find the de Broglie wavelength of a π^+ meson of kinetic energy 100 MeV. Compare it with the wavelength of a photon of the same total energy. [6.4fm, 5.2fm]

2. What is the de Broglie wavelength for an average helium atom in a furnace at 400K ? [66 pm]

3. A particle has a mass of 1kg. How long does it take to move through a distance of 1m if its de Broglie wavelength is comparable to that of visible light (500nm)? What is the corresponding answer if the particle is an electron ? [2.3×10^{19}years, 6.8×10^{-4}s]

4. The position and momentum of a 1keV electron are simlutaneously determined. If its position is located to within 0.1nm, what is the minimum percentage uncertainty in its momentum? [$(\Delta p \Delta x \approx h \quad 38\%)$] or [$(\Delta p \Delta x \approx \hbar \quad 38\%)$]

5. A 50g bullet travels at 333ms^{-1} (accurate to 0.01%). Show that for practical purposes, the Uncertainty Principal poses no limitations on hitting the target.

6. Show that the ground state energy of a particle in a rectangular linear potential well with infinitely high sides is compatible with Heizenberg's Uncertainty Principal.

7. A linear quantum dot can be modelled as a electron in a box. It can be formed by deposting contacts on a slab of GaAs and then applying a bias voltage in such a way as to create a linear confining potential. Suppose the observed ground-state electron energy is 0.03 eV. How big is the quantum dot ? [5.7nm]

8. Find the ground state energy and the first two excited states for an electron in a one dimensional box of atomic dimensions (1Å). By contrast, what is the groundstate energy of a 10^{-2} kg marble in a 0.1m wide box. If the marble rolls in the box with a very slow velocity of 10^{-2} m/year, what is its corresponding quantum number ?
[38eV, 151eV, 340eV, 3.4×10^{-45}eV, $n = 9.5 \times 10^{-45}$]

9. Use Schrödinger's equation to solve the problem of an infinitely deep square well with the origin at the center of the well and $V = 0$ for $-\frac{a}{2} \leq x \leq \frac{a}{2}$. Show that the solutions are alternatively of even and odd parity. (Even parity $\Rightarrow f(x) = f(-x)$, odd parity $\Rightarrow f(x) = -f(-x)$.)

10. A particle is confined between rigid walls a distance L apart. What is the probability that it will be found within a distance $L/3$ of one wall

    (a) when the particle is in its ground state,

    (b) when it is in the $n = 2$ state and

    (c) under the assumption of classical physics ($n = \infty$).e

    (Hint: $\int \sin^2 x = \frac{1}{2}(x - \cos x \sin x)$.) [0.2, 0.4, 0.33]