Tutorial 4 : Statistical Mechanics

1. At the same temperature, will a gas of classical molecules, a gas of bosons, or a gas of fermions exert the greatest pressure ? The least pressure ? Why ?

2. Show that the average kinetic energy per molecule at room temperature is much less than the energy needed to raise the hydrogen atom from its ground state to its first excited state.

3. A red star and a white star radiate energy at the same rate, can they be the same size ? If not, which must be the larger ? Explain.

4. A thermograph is an instrument that maps the rate at which each small portion of a person's skin emits infrared radiation. To verify that a small difference in skin temperature corresponds to a significant difference in radiation emission rate, find the percentage difference between the total radiation from the skin at 34° and at 35°C. [1.3%]

5. At what rate would solar energy arrive at the earth if the solar surface temperature were 10% lower that its current value of 5800°C.
[0.865 kW/m^2]

6. The microprocessors used in computers produce heat at a rate as high as 30W per square centimeter of surface area. What temperature would it be at if it had such a radiance with no conductive or convective cooling ? [1516 K]

7. Human skin temperature is about 34°C. What is the wavelength of maximum spectral radiance ? [9.35μm - infrared]

8. What is the connection between the fact that free electrons in a metal obey Fermi statistics and the fact that the photo-electric effect is virtually temperature independent ?

9. Find the median energy in a free electron gas at $T = 0$.
[$\epsilon_F/2^{2/3} = 0.630\epsilon_F$]

10. The Fermi energy in copper is 7.04 eV. Compare the approximate average energy of electrons in copper at room temperature (T=0.025 eV) with what their average energy would have been as classical particles.
[4.22 eV, 37.5 meV]

11. Show that if the average occupancy of a state of energy $\epsilon_F + \Delta\epsilon$ is f_1 at any temperature, then the average occupancy of a state of energy $\epsilon_F - \Delta\epsilon$ is $1 - f_1$.
(This explains the symmetry of the curve for the Fermi distribution.)

12. The density of metallic zinc is 7.13 g/cm^3 and the atomic mass of the zinc atom is 65.4 u. The Fermi energy in zinc metal is 11.0 eV. Work out the effective mass of a delocalised electron in Zinc metal. Express your answer in terms of the free electron mass. Why is there a difference ?
(The electronic structure of atomic zinc may be found in a reference book.) [0.85m_e]

13. Find the number of electron states per electron volt at $\epsilon = \epsilon_F/2$ in a 1.00 gram sample of copper at $T = 0$ K. Compare this with the number of electron states per electron volt for a metallic cluster of 1000 copper atoms. Comment of the classical versus the quantum mechanical behaviour of expected for these two situations.

[1.43×10^{21} /eV, 150 /eV]

14. The gravitational potential energy of a star is $E_g = -\frac{3}{5}GM^2/R$. Consider a particular white dwarf star consisting mostly of carbon, (typical for a star terminating its life with the carbon fusion cycle). Write down the number of electrons N for this star in terms of its mass. Next consider that the star has a radius given by hydrostatic equilibrium between the gravitational and Pauli pressure. Assume the electrons generate the dominant contribution to the Pauli pressure. If the star has cooled so that $kT \ll \epsilon_F$ then the average electron energy is $\frac{3}{5}\epsilon_F$ and the Pauli energy is $E_e = \frac{3}{5}N\epsilon_F$. The total energy of the star is then $E = E_g + E_e$. Find the equilibrium radius R of the star. Considering a star half the mass of the sum, find its radius.

(Hint : set $dE/dR = 0$.) [$R = \frac{Nh^2}{m_e}\left(\frac{9N}{32\pi^2}\right)^{3/2}\frac{1}{GM^2}$, 36 m]