Tutorial Questions: Lasers

Question 1

Assume a two level laser system with no degeneracy ($g_1 = g_2 = 1$).

(a) If $N_2 + N_1 =$ constant, show that $B_{12} = B_{21}$.
(b) At what wavelength is the coefficient of spontaneous emission equal to the coefficient of stimulated emission?
(c) Derive an expression to explain why materials with long excited state lifetimes are easier to get to laser than those with short lifetimes (hint: solve for $A_{21}(t)$ and then $B_{21}(t)$). Plot the decay of the upper laser level, $N_2(t)$, for the three lasing materials discussed in the lectures (Nd:YAG, Nd:$GdVO_4$ and Nd:YLF). Assume $N_2(0) = 100$. Which material do you think would have the highest optical gain? Explain your answer.
(d) In a Ruby laser $\lambda \sim 694$ nm. Show why this laser cannot operate when the gain material is in thermal equilibrium at T = 300K. (Hint: use Boltzmann's statistics).

Question 2

Calculate the average power, peak power and fluence of the following laser beams:

(a) 10Hz operation of a Nd:YAG laser at 1064nm, with 100mJ per pulse in a 20ns time envelope, focussed to a 100μm spot size.
(b) 100Hz operation of a CO_2 laser at 10.6μm, with 1J per pulse in a 2μs time envelope, focussed to a 500μm spot size.
(c) 80MHz operation of a mode locked Ti:Sapphire laser at 780nm , with 1nJ per pulse in a 50fs time envelope, focussed to a 20μm spot size.

Question 3

(a) What must the waist of a HeNe laser be for a half angle divergence of 1mrad?
(b) What would the Rayleigh range of such a beam be?
(c) If this beam was focussed with a lens of focal length 100mm, what would the beam radius be at the focal plane of the lens?
(d) Why does the answer in (c) not depend on the position of the beam waist prior to the lens?

Question 4

(a) Calculate the beam size on the moon from an Ar:Ion laser ($\lambda = 514.5$nm) if the laser waist is 25mm radius, and is located at the output coupler of the laser beam. Assume the laser output is Gaussian, and that the moon is 385 000km away.
(b) What would the initiate waist radius have to be to have a beam radius of 100m at the moon?

Question 5

The propagation of a laser beam after a lens is given by the following:

$$w^2(z) = \left(1 - \frac{z}{f}\right)^2 w_o^2 + \left(z + \left(1 - \frac{z}{f}\right)d\right)^2 \theta_o^2$$

where w_o and θ_o are the beam waist and divergence prior to the lens (of focal length f), d is the distance from w_o to the lens, and z is the distance from the lens.

(a) Derive an equation for the waist position after the lens.
(b) Show under what conditions the geometric approximations will be true (i.e., that the new waist position is at the focal plane of the lens).

Question 6

(a) The ruby laser was the first laser demonstrated. It is flash lamp pumped (range of wavelengths), and then emits light in the red. Why does this simple fact immediately show that its lasing mechanism must involve more than two energy levels?
(b) How many optical cycles would the pulse envelope cover of a 100fs laser at 780nm?

Question 7 (2005 exam question)

(a) Show that the transmission of a Gaussian laser beam of $1/e^2$ radius w, through a circular aperture of radius a, can be written as:

$$T = T_0\left(1 - \exp\left(-\frac{2a^2}{w^2}\right)\right)$$

where T is the fraction of energy transmitted through the aperture, and T_0 is the maximum transmission.

(6)

(b) The table of data given below was collected for a CO_2 laser beam passing through a circular aperture. Use the data to estimate the $1/e^2$ beam radius w assuming that the laser beam has a Gaussian fluence profile.

Aperture diameter (mm)	**Transmitted energy (mJ)**
1.5	71
3	244
4.0	368
5.0	428
6.0	468
∞	488

(3)

(c) Lasers usually emit in a range of closely spaced wavelengths satisfying the standing wave condition: $m\lambda/2 = L$, where m is an integer and L is the cavity length. Show that the smallest possible interval between wavelengths can be approximated as:

$$\Delta\lambda \approx \lambda^2 / 2L$$

(2)

(d) An ultra-short pulsed laser outputs 10nJ pulses of FWHM duration 50fs at 780nm wavelength. If the resonator cavity length is 1m, how many longitudinal modes would have to be oscillating? What would the gain bandwidth of this laser have to be to support these modes? At what repetition rate would you expect this laser to operate at? What would the peak power and average power of the laser be?

(5)